Hyperbolic manifold is an "ordinary" manifold with curvature -1.

There is a formula for computing the volume of certain polyhedra in hyperbolic space, it can be expressed in terms of Lobachevsky function

$$\Lambda(\theta) = -\int_0^\theta \log(2\sin u)\, du$$

The volume of the hexahedron

$$V_{hexahedron} = \Lambda\left(\frac{\theta_1}{2}\right) + \Lambda\left(\frac{\theta_2}{2}\right)$$

The volume of the tetrahedron

$$V_{tetrahedron} = \Lambda(\theta_1) + \Lambda(\theta_2) + \Lambda(\theta_3)$$

Milnor's formulas see Hyperbolic geometry: the first 150 years

There is an interesting observation: one can obtain this expression from the star-triangle relation for the integrable models arXiv: hep-th/0703041

The idea to understand the relationship between hyperbolic geometry (in terms of volume computations) and integrable models of lattice spin models of statistical mechanics.

Papers:

- E. Vinberg, Volumes of non-Euclidean polyhedra, 1993
- J. Milnor, Hyperbolic geometry: the first 150 years, 1982
- V. Bazhanov, V. Mangazeev, S. Sergeev, Faddeev-Volkov solution of the Yang-Baxter equation and discrete conformal symmetry arXiv: hep-th/0703041, 2007