1. For $a, b > 0$ show that

$$\sqrt[3]{ab^2} \leq \frac{a+2b}{3}$$

2. For $a > 0$ show that

$$a + \frac{1}{a} \geq 2$$

3. Solve the following equation

$$x^4 + ax^3 + bx^2 - ax + 1 = 0$$

Hint: use change of variable

4. Show that among all rectangles having the same perimeter, the square has the maximum size.

5. Prove the arithmetic and geometric means inequality for 4 numbers

$$\frac{a_1+a_2+a_3+a_4}{4} \geqslant \sqrt[4]{a_1 \cdot a_2 \cdot a_3 \cdot a_4}$$

6. Express $(x_1-x_2)^2$ in terms of p and q if x_1 and x_2 are roots of the equation

$$x^2+px+q=0$$

7. Express $(x_1-x_2)^2(x_2-x_3)^2(x_1-x_3)^2$ in terms of p, q and r if x_1, x_2 and x_3 are roots of the equation

$$x^3+px^2+qx+r=0$$

8. Reduce the following quadratic form to the canonical form

$$f = 4x_1^2 + 2x_2^2 + x_3^2 + x_4^2 - 4x_1x_2 - 4x_1x_3 + 4x_1x_4 + 4x_2x_3 - 4x_3x_4$$

9. Find the rank of quadratic form

$$5x_1^2 + 2x_1x_2 - 6x_1x_3 + x_2^2 - 2x_2x_3 + 2x_3^2$$

10. $a_1, a_2, \ldots, a_n$ are nonnegative numbers and $a_1 + a_2 + \ldots + a_n = a$. Show that

$$a_1a_2 + a_2a_3 + \ldots + a_{n-1}a_n \leq \frac{a^2}{4}$$

(11) Solve the following equation

$$\frac{1}{x^2} - \frac{1}{(x+1)^2} = 1$$

(12) If $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{a+b+c}$, prove that the equality

$$\frac{1}{a^n} + \frac{1}{b^n} + \frac{1}{c^n} = \frac{1}{a^n+b^n+c^n}$$

holds for any odd n.

(13) If $|ax^2+bx+c| \leq \frac{1}{2}$ for $|x| \leq 1$ show that $|ax^2+bx+c| \leq x^2 - \frac{1}{2}$ for $|x| \geq 1$.

(14) If $|ax^2+bx+c| \leq 1$ for $|x| \leq 1$, show that $|cx^2+bx+a| \leq 2$ for $|x| \leq 1$.

(15) Let α be a root of the equation $x^2+ax+b=0$ and β be a root of equation $x^2-ax-b=0$. Prove that the equation $x^2-2ax-2b=0$ has a root between the numbers α and β.