1. Solve the following equation

$$3 - 7\cos^2 x \sin x - 3\sin^3 x = 0$$

2. Prove that the complex number

$$\frac{z-1}{z+1}, \quad z \neq \pm 1$$

is pure imaginary (i.e. real part is zero) if and only if $|z| = 1$.

3. Find equation of straight line $y = 3x - 2$ in complex plane.

4. Show that the following equality holds

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$$

for any complex numbers z_1 and z_2. What is geometric meaning of this equality?

5. Solve the equation

$$z^4 = \bar{z}^4$$

6. Solve the equation

$$x^4 + x^3 + x^2 + x + 1 = 0$$

7. Show the following equality

$$x^{2n} + 1 = \prod_{j=0}^{n-1} \left(x^2 - 2x \cos \frac{(2j+1)\pi}{2n} + 1 \right)$$

↑ symbol for product:

$$\prod_{j=0}^{n} a_j = a_0 \cdot a_1 \cdot a_2 \cdot \ldots \cdot a_n$$

8. If $z + \frac{1}{z} = 2\cos\alpha$, show that

$$z^n + \frac{1}{z^n} = 2\cos n\alpha$$

9. Using complex numbers show the following equality

$$\arctan 1 + \arctan \frac{1}{2} + \arctan \frac{1}{3} = \frac{\pi}{2}$$

10. Solve the equation

$$\cos z = \frac{5}{4} \qquad z \in \mathbb{C}$$

Hint: use the definition of cosine in terms of exponential.

11. Solve the equation

$$x^4 + ax^3 + (a+b)x^2 + 2bx + b = 0$$

12. Solve the equation

$$(x^2 - x - 1)^3 + (x^2 - 3x + 2)^3 = (2x^2 - 4x + 1)^3$$

13. Find all n for which the following system holds

$$x_1 + x_2 + \ldots + x_n = 3$$

and

$$\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n} = 3$$

where $x_1, x_2, \ldots, x_n > 0$.