

Problem 1.

x_1 and x_2 are roots of quadratic equation $x^2 + px + q = 0$. Show that

$$\frac{x_1^n + x_2^n}{n} = \sum_m (-1)^{n+m} \frac{(n-m-1)!}{m!(n-2m)!} p^{n-2m} q^m$$

Hint: Use mathematical induction.

Problem 2. Solve the equation

$$1 - \frac{x}{1} + \frac{x(x-1)}{2!} - \dots + (-1)^n \frac{x(x-1)\dots(x-n+1)}{n!} = 0$$

Problem 3. Find all roots of the equation

$$\cos x + \cos 2x + \cos 3x + \frac{1}{2} = 0$$

Problem 4. The polynomial $P(x)$ for all real x takes only positive values. Prove that there are polynomials $a(x)$ and $b(x)$ for which $P(x) = a^2(x) + b^2(x)$

Problem 5.

All roots of the equation $z^3 + az^2 + bz + c = 0$ lie on a circle $|z| = 1$. Show that all roots of the equation $z^3 + |a|z^2 + |b|z + |c| = 0$ also lie on a circle $|z| = 1$.