Problem 1. A is a $2n \times 2n$ matrix such that all diagonal elements are zero, $A_{ii} = 0$ and all other entries equal to $+1$ or -1. Prove that $\det A \neq O$.

Problem 2. Let A be an $n \times n$ matrix with entries $A_{ij} = \max(i, j)$. Compute the determinant of this matrix.

Problem 3. For $x^2 + y^2 + z^2 = 1$ prove the following inequality

$$\det \begin{pmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{pmatrix} < 1$$

Problem 4. Compute the following determinant

$$\det \begin{pmatrix} 0 & 1 & 1 & 1 & \dots & 1 \\ 1 & 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & \frac{1}{2} & 0 & \dots & 0 \\ 1 & 0 & 0 & \frac{1}{3} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & 0 & \dots & \frac{1}{n} \end{pmatrix}$$

Problem 5. Let A be an $n \times n$ matrix with entries $A_{ij} = \sum_{k=1}^{n} k^{i+j}$. Compute the determinant of this matrix.

Hint: Use the Vandermonde matrix.