PHYS301: Midterm II

October 30, 2022

Problem 1. Write the expression for the conserved angular momentum in a central force problem.

Problem 2. Make the coordinate transformation $q = e^{-\alpha t/2}x$ in the Lagrangian

$$L(q, \dot{q}, t) = \frac{1}{2}e^{\alpha t}(\dot{q}^2 - \omega^2 q^2)$$

and find the equation of motion in terms of x.

Problem 3. A particle of mass m is constrained to move on an infinite homogeneous cylindrical helix without the influence of any other external field. Find an integral of motion via the Noether theorem.

Problem 4. Given the internal energy $dU = TdS - PdV$, determine the enthalpy $H(S, P)$ and the Gibbs free energy $G(T, P)$.

Problem 5. Consider a two-dimensional pendulum of length l with mass m at its end which is swinging under the influence of gravity (spherical pendulum). Write the Lagrangian and the Euler-Lagrange equation for this system.

Problem 6. Write the Hamiltonian and the Routhian for a spherical pendulum.

Problem 7. Show that the shortest trajectory connecting two points on the surface of a sphere is an arc of a great circle.

Problem 8. Solve the equation of motion with initial conditions $x(0) = 1$ and $\dot{x}(0) = 0$ for a system with the following Lagrangian

$$L = \dot{x}^2 - \frac{1}{x}$$