

Problem 1.

a , b and c are complex numbers.

Show that the following inequalities are equivalent to each other

$$\operatorname{Re}((a-c)(\bar{c}-\bar{b})) \geq 0$$

and

$$\left| c - \frac{a+b}{2} \right| \leq \frac{1}{2} |a-b|$$

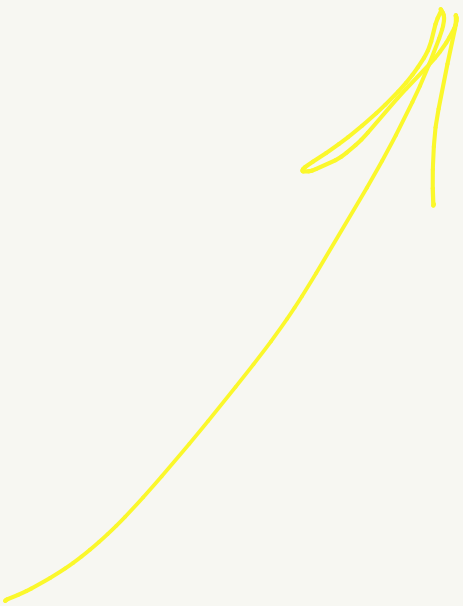
Problem 2.

Show that

$$\underbrace{x^{2n} - 1}_{= (x^2 - 1)} = (x^2 - 1) \left(x^2 - 2x \cos \frac{\pi}{n} + 1 \right)$$

$$\cdot \left(x^2 - 2x \cos \frac{2\pi}{n} + 1 \right) \dots$$

$$\cdot \left(x^2 - 2x \cos \frac{(n-1)\pi}{n} + 1 \right)$$


$$x^{2n-1} - 1 = (x^2 - 1) \left(x^2 - 2x \cos \frac{\pi}{n} + 1 \right) \left(x^2 - 2x \cos \frac{2\pi}{n} + 1 \right) \left(x^2 - 2x \cos \frac{3\pi}{n} + 1 \right) \dots \left(x^2 - 2x \cos \frac{(n-1)\pi}{n} + 1 \right)$$