

Problem 1. Show that

$$\sum_{k=1}^n f_k^2 f_{k+1} = \frac{1}{2} f_n f_{n+1} f_{n+2}$$

where f_k is the k 'th Fibonacci number.

Problem 2. Let

$$(1 + \sqrt{2} + \sqrt{3})^n = a_n + b_n \sqrt{2} + c_n \sqrt{3} + d_n \sqrt{6},$$

$\sqrt{2} \cdot \sqrt{3}$
↓
 $\sqrt{6}$

where a_n, b_n, c_n and d_n are natural numbers. Find the following limits

$$\lim_{n \rightarrow \infty} \frac{b_n}{a_n}, \quad \lim_{n \rightarrow \infty} \frac{c_n}{a_n}, \quad \lim_{n \rightarrow \infty} \frac{d_n}{a_n}$$