PHYS302: Final

March 20, 2023

Problem 1. Find the frequency of small oscillations of the following system

$$L = \cosh x (\dot{x}^2 - 1) \ .$$

Problem 2. Express the components of the mass quadrupole moment tensor,

$$D_{ij} = \int (3x_i x_j - r^2 \delta_{ij}) \rho dV \ ,$$

where ρ is the density, in terms of the components I_{ij} of the inertia tensor.

Problem 3a. Find the eigenfrequencies of coupled harmonic oscillators with equal masses m, all with spring constant k, with the system fixed at the ends. Note that the motion can only take place in one dimension.

Problem 3b. Write the equations of motion for the symmetric top, i.e. for a rigid body with inertia I_3 around a symmetry axis, and I_1 around the two perpendicular principle axes of rotation.

Problem 4. Find the frequency of the small oscillations for the system depicted in the Figure. The end of the system moves on a vertical axis and the whole system rotates with the angular velocity in the field of gravity around a vertical axis and $\omega > \sqrt{\frac{2g}{a}}$.