Problem 1. Find the change in energy levels of a charged quantum linear oscillator in a uniform electric field applied along the oscillation axis

$$U = \frac{kx^2}{2} - e\varepsilon x,$$

where ε is an electric field.

Problem 2. In a region of space, a quantum particle with mass m and with zero energy has the following wave function

$$\psi(x) = Cxe^{-x^2/d^2}$$

where C and d are constants. Find the potential energy $U(x)$ of the particle.

Problem 3. Find the discrete energy spectrum and wave-functions for the following potential

$$U(x) \;=\; \frac{U_1}{(1+e^{x/a})^2} - \frac{U_2}{1+e^{x/a}}\;,$$

where $U_1, U_2, a > 0$ are some constants and $U_1 > \frac{1}{2}U_2$.