Problem 1. Show that the Poisson bracket of any two integrals of motion I_1 and I_2 of a system, which are explicitly independent of t, is itself an integral of motion of the system.

Problem 2. Show that the Poisson bracket

$$\{J_3,\ J_1^2 + J_2^2 + J_3^2\} = 0,$$

where J_i are the components of the angular momentum vector.

Problem 3. Show that the determinant of the Lax operator is an integral of motion of the system.

Problem 4. Show that for the isospectral problem of the Schrödinger operator with the evolution operator

$$A_1 = \alpha \frac{d}{dx} + \beta,$$

the potential u satisfies

$$u_t = \alpha u_x.$$