Problem 1. Show that the Euler-Lagrange equation can be written in the following form

$$\frac{d}{dt}\left(L - \dot{q}\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial t} = 0\ .$$

Problem 2. Show that the Euler-Lagrange equation is a second order ordinary differential equation.

Problem 3. Write the following Lagrangian in a simpler form

$$L = \frac{1}{2}\left(\dot{q} + q\right)^2\ .$$

Problem 4. Show that the Euler–Lagrange equation obtained from the Lagrangian

$$\bar{L}(x, \dot{x}) \ = \ \frac{1}{12}\dot{x}^4 \ + \ \frac{\omega^2}{2}x^2\dot{x}^2 \ - \ \frac{\omega^4}{4}x^4$$

possesses the same set of solutions as the equation of motion for a harmonic oscillator with unit mass.

Problem 5. Given a Lagrangian of the form $L(q_i, \dot{q}_i, \ddot{q}_i, t)$ show that Hamilton's principle, with $\delta q_i = \delta \dot{q}_i = 0$ at the endpoints, gives rise to the following generalised form of Lagrange's equations:

$$\frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{q}_i}\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) + \frac{\partial L}{\partial q_i} = 0.$$