Problem 1. Compute the action corresponding to the classical motion of a free particle.

Problem 2. A system is described by the Lagrangian

$$L = \frac{\dot{x}^2}{2} + t^2\dot{x}.$$

Write down the Euler–Lagrange equation and find $x(t)$.

Problem 3. A system is described by the Lagrangian

$$L = -\sqrt{1 - v^2} + Av - B,$$

where A and B are functions of the coordinates. Find the energy of the system.

Problem 4. Show that Lagrangians differing by the total time derivative of any function of the coordinates and time lead to the same Euler–Lagrange equations.