Problem 1. Write the Lagrange function for a particle in the following coordinate systems

(1) Cartesian coordinates (x, y, z),
(2) Cylindrical coordinates (ρ, ϕ, z),
(3) Coordinates (u, v) related to Cartesian coordinates by

$$x = u - \frac{v^2}{2}, \quad y = uv.$$

Problem 2. Write the Lagrange function for a particle

(1) in a uniformly moving coordinate system;
(2) in a uniformly accelerated coordinate system;
(3) in a uniformly rotating coordinate system.

Problem 3. The Lagrangian of a free particle has the following form

$$L = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$$

The Galilean transformation

$$x' = x, \qquad y' = y - 3t, \qquad z' = z + 5t, \qquad t' = t$$

does not leave the form of the action invariant. Nevertheless, this transformation corresponds to a certain integral of motion. Determine this integral using the generalized Noether theorem.

Problem 4. The Lagrangian of the system is given by

$$L = \dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2 - U(x_1 - x_2) - U(x_1 - x_3) - U(x_2 - x_3),$$

where

$$U(x) = \frac{1}{a^2 x^2}.$$

Show that the quantity

$$I = 2\dot{x}_1\dot{x}_2\dot{x}_3 - \dot{x}_1 U(x_2 - x_3) - \dot{x}_2 U(x_1 - x_3) - \dot{x}_3 U(x_1 - x_2)$$

is an integral of motion.