**Problem 1.** Consider two particles with masses $m_1$ and $m_2$ interacting through a potential $U(|r_1 - r_2|)$. The Lagrangian of the system is given by

$$L = \frac{m_1 \dot{r}_1^2}{2} + \frac{m_2 \dot{r}_2^2}{2} - U(|r_1 - r_2|).$$

Rewrite the Lagrangian in terms of the relative coordinate $r = r_1 - r_2$ and the center-of-mass coordinate $R$.

**Problem 2.** Find the Hamiltonian of the system with the Lagrangian

$$L = \frac{\dot{x}^2}{x} + x + \frac{1}{x}.$$

**Problem 3.** Find the period of the system described by the Lagrangian

$$L = \dot{x}^2 - \frac{2}{x} - \frac{1}{x^2}.$$

**Problem 4.** The Lagrangian of the system is

$$L = \dot{r}^2 + r^2\dot{\theta}^2 + r^2\dot{\varphi}^2 \sin^2\theta - 4\dot{\varphi}\cos\theta.$$

Verify that the quantity

$$I = \mathbf{r} \times \dot{\mathbf{r}} - 2\frac{\mathbf{r}}{r}$$

is an integral of motion.