**Problem 1.** Find the accelerations if the Lagrangian function is given as follows:

(a) $L = \dfrac{\dot{q}^2}{2} - \dfrac{q^2}{2}$; (b) $L = \dfrac{(1+q^2)\dot{q}^2}{2} - \dfrac{q^2}{2}$; (c) $L = \dfrac{t\dot{q}^2}{2}$;

(d) $L = -\sqrt{1-\dot{q}^2} + q$; (e) $L = \dfrac{\dot{r}^2}{2} + \dfrac{r^2\dot{\theta}^2}{2} + \dfrac{1}{r}$; (f) $L = \dfrac{\dot{\theta}^2}{2} + \dfrac{\sin^2\theta\,\dot{\varphi}^2}{2} - \cos\theta$.

**Problem 2.** Simplify the Lagrangian functions by excluding total derivatives:

(a) $L' = \dot{x}\sin t$, (b) $L' = \dfrac{1}{2}(\dot{q}+t)^2$,

(c) $L' = \dfrac{1}{2}(\dot{q}+q)^2$, (d) $L' = x\dot{y} - y\dot{x}$.

**Problem 3.** Integrate the equations of motion of a particle if its Lagrangian function and the initial conditions are given (at the moment $t=0$: velocity and coordinate $\dot{x} = \dot{x}_0,\ x = x_0$):

(a) $L = \dot{x}^2 - \dfrac{1}{x^2}$, $x_0 = 1,\ \dot{x}_0 = 0$,

(b) $L = \dot{x}^2 + \tan^2 x$, $x_0 = 0,\ \dot{x}_0 = 2$,

(c) $L = \dot{x}^2 + e^x$, $x_0 = 0,\ \dot{x}_0 = 2$,

(d) $L = \dot{x}^2 - \dfrac{1}{x}$, $x_0 = 1,\ \dot{x}_0 = 0$.

**Problem 4.** Write down the Hamilton's equations if the Hamiltonian function is given as follows:

(a) $H = \dfrac{p_r^2}{2} + \dfrac{p_\theta^2}{2r^2\sin^2\theta} + \sin\theta$; (b) $H = \dfrac{p_u^2 + p_v^2}{2(u^2+v^2)} + u$;

(c) $H = \sqrt{1+p^2} + r$; (d) $H = \dfrac{p_x p_y}{t x^2 y}$.

**Problem 5.** Find the Hamiltonian function if the Lagrangian function is given as follows:

(a) $L = \dfrac{m}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - U(x,y,z)$; (b) $L = \dfrac{m}{2}(\dot{r}^2 + r^2\dot{\varphi}^2 + \dot{z}^2) - U(r,\varphi,z)$;

(c) $L = \dfrac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\varphi}^2) - U(r,\varphi,\theta)$; (d) $L = -mc^2\sqrt{1 - \dfrac{v^2}{c^2}}$.