Problem 1. Consider a harmonic oscillator with angular frequency ω, namely $x = A\cos(\omega t + \varphi)$.

(a) The instantaneous position x and velocity $\dot{x}$ of the particle are given. Determine the amplitude A and the phase φ of the motion.

(b) Express the amplitude A and the phase φ in terms of the position x and the total energy E of the oscillator.

Problem 2. For each of the following Lagrangians $L(x, \dot{x})$, determine the equilibrium point(s) and compute the frequency of small oscillations about equilibrium.

(a)

$$L = \frac{m\dot{x}^2}{2} - \frac{kx^2}{2};$$

(b)

$$L = \frac{\dot{x}^2}{2} + \sin x;$$

(c)

$$L = \frac{\dot{x}^2}{x} - \frac{x}{\ln x}.$$

Problem 3. Consider a linear harmonic oscillator governed by

$$\ddot{x}(t) + \omega^2 x(t) = F(t),$$

where ω is the natural angular frequency of the free oscillator. Assume zero initial conditions:

$$x(0) = 0, \qquad \dot{x}(0) = 0.$$

For each of the following external forces $F(t)$, determine the motion $x(t)$:

(a) $F(t) = at$

(b) $F(t) = ae^{-\alpha t}$

(c) $F(t) = ae^{-\alpha t}\cos(\beta t)$