Problem 1. Consider a one-dimensional chain of identical point masses m connected by springs. The masses are arranged at equal equilibrium distances a. The spring constants alternate along the chain and take two values k_1 and k_2:

$$k_n = \begin{cases} k_1, & n \text{ even}, \\ k_2, & n \text{ odd}. \end{cases}$$

Here k_n denotes the spring constant between the masses n and $n+1$. Let $x_n(t)$ denote the small displacement of the n-th mass from equilibrium.

(1) Write down the equations of motion for the displacements $x_n(t)$.
(2) Assume solutions in the form of normal modes.
(3) Show that the frequencies $\omega(q)$ satisfy the dispersion relation

$$\omega_{\pm}^2(q) = \frac{k_1 + k_2}{m} \pm \frac{1}{m}\sqrt{k_1^2 + k_2^2 + 2k_1 k_2 \cos(a/2)}.$$