**Problem 1.** Let a system of point masses $m_a$ be located at positions

$$\mathbf{r}_a = (x_{a,1}, x_{a,2}, x_{a,3}),$$

and define the quadrupole moment tensor by

$$D_{ij} = \sum_a m_a \left(3x_{a,i}x_{a,j} - r_a^2\delta_{ij}\right),$$

where

$$r_a^2 = x_{a,1}^2 + x_{a,2}^2 + x_{a,3}^2.$$

The inertia tensor of the same system is defined as

$$I_{ij} = \sum_a m_a \left(r_a^2\delta_{ij} - x_{a,i}x_{a,j}\right).$$

Show that the quadrupole moment tensor can be written in terms of the inertia tensor

**Problem 2.** The Hamiltonian of the Euler top in Andoyer variables

$$h,\ g,\ l,\ p_h,\ p_g,\ p_l$$

is given by

$$H = \frac{p_g^2 - p_l^2}{2}\left(\frac{\sin^2 l}{I_1} + \frac{\cos^2 l}{I_2}\right) + \frac{p_l^2}{2I_3}.$$

Here $h$, $g$, and $l$ are canonical coordinates, and $p_h$, $p_g$, $p_l$ are the corresponding conjugate momenta. The constants $I_1$, $I_2$, and $I_3$ are the principal moments of inertia.Derive the Hamilton equations for the special case $I_1 = I_2$ and solve them.

**Problem 3.** Three point masses, each of mass $m$, are located at the points

$$(a, 0, 0), \qquad (0, a, 0), \qquad (a, a, 0).$$

Compute the inertia tensor of the system with respect to the origin.