

**Problem 1.** Consider the Lagrangian

$$L = \frac{1}{2}\dot{x}_1^2 - \dot{x}_2x_3.$$

Using the Dirac-Bergmann formalism, perform the following steps

- (1) Show that the Lagrangian is singular by computing the Hessian matrix

$$\frac{\partial^2 L}{\partial \dot{x}_i \partial \dot{x}_j}$$

and verifying that its determinant vanishes.

- (2) Determine all primary constraints of the system.  
(3) Construct the canonical Hamiltonian

$$H_c = \sum_{i=1}^3 p_i \dot{x}_i - L,$$

expressed in terms of phase-space variables.

- (4) Write down the total Hamiltonian

$$H_T = H_c + \sum_a \lambda_a \phi_a,$$

by adding the primary constraints  $\phi_a$  with Lagrange multipliers  $\lambda_a$ .